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Indefinite Charge for the Classical Spinor Field

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Abstract

We define a conserved Lorentz vector for a two-component spinor field that obeys the Klein-Gordon equation and interpret it as a charge-current density. The corresponding total charge can take negative as well as positive values, which is not the case for the usual charge of the Dirac field. We consequently can define probability amplitudes for a relativistic quantum mechanics, and we solve the inhomogeneous equation by means of the causal Green function. This vector is not invariant under gauge transformations of the spinor field, and we cannot generalize the equation by the gauge invariant substitution to obtain the interaction with an electromagnetic field. In the limit of a massless field that obeys the Weyl equation, the charge vanishes.

1. *Introduction*

The Klein-Gordon equation

$$(\partial^2 + m^2)\phi(x) = 0 \tag{1.1}$$

for the complex scalar field ϕ and the Dirac equation

$$(-i\gamma \cdot \partial + m)\psi(x) = 0 \tag{1.2}$$

for the bispinor field ψ were proposed as relativistic generalizations of the Schrödinger equation in quantum mechanics. The conserved four-vector

$$j_\mu^{(\mathrm{KG})} = i(\phi^* \phi_{,\mu} - \phi^*_{,\mu}\phi) \tag{1.3}$$

was found unsuited for a probabilistic interpretation because j_0 is not positive. On the other hand,

$$j_\mu^{(\mathrm{D})} = \bar{\psi}\gamma_\mu\psi \tag{1.4}$$

has a positive j_0, which was considered an advantage of the Dirac equation over the Klein-Gordon equation. These vectors were later interpreted in terms of charge and current densities, and relativistic quantum mechanics was abandoned in favor of quantum theory of fields.

We pointed out (Marx, 1969) how a probabilistic interpretation could be extended to the relativistic theory of a charged scalar particle interacting with an external electromagnetic field. Use of the causal Green-function or Feynman propagator leads to the specification of initial and final conditions. The interpretation of the wave function in terms of probability amplitudes was a direct consequence of charge conservation, and it allowed for a proper account of pair creation and annihilation. We extended the theory to several particles through Dirac's many-times formalism (Marx, 1970a).

Thus, the positive j_0 of the Dirac equation becomes a drawback. We have modified this equation along the lines of quantum field theory (Marx, 1970b, 1972), by introducing an observer-dependent Lagrangian density (Marx, 1970c), or a third-order equation (Marx, 1974). However, these approaches do not lead to a truly satisfactory formulation of relativistic quantum mechanics of spin-$\frac{1}{2}$ particles.

Here we show that the Dirac field possesses a conserved Lorentz vector with an indefinite charge density. We use the two-component spinor form of the Dirac equation (Feynman and Gell-Mann, 1958), which for the free field reduces to the Klein-Gordon equation. A Lagrangian density constructed in a similar manner vanishes identically, and other tensor densities have the same problem.

In Section 2 we find the momentum-space expansion of the solutions and compute the conserved charge. We define probability amplitudes for particles and antiparticles, and consider the invariance of the equation under space reflection, time reflection, and charge conjugation. In Section 2 we use the causal Green-function to find the solution of the inhomogeneous equation in terms of integrals over the sources and appropriate "boundary" conditions. We briefly discuss the massless field in Section 4.

The charge and current density vector is *not* invariant under gauge transformations of the first kind, which precludes the introduction of electromagnetic interactions through the usual gauge-invariant substitution. These difficulties are discussed in Section 5.

The notation in this paper follows that used in previous ones (Marx, 1970c, 1974). We use a time-favoring metric, natural units, and the modified summation convention for repeated lower Greek indices.

2. *The Free Field*

We make the customary assumption that the free spinor field obeys the Klein-Gordon equation

$$(\partial^2 + m^2)\chi_A(x) = 0 \tag{2.1}$$

Under a proper orthochronous Lorentz transformation, the two-component spinor field transforms according to the equation

$$\chi'_A = s_A{}^B \chi_B \tag{2.2}$$

where s is a unimodular 2 x 2 matrix, that is, its determinant is 1. The complex conjugate field thus transforms with s^*, and we indicate this by using dotted indices, that is,

$$[\chi_A(x)]^* = \chi^*_{\dot{A}}(x) \tag{2.3}$$

Consequently, a quantity such as $\chi^*_{\dot{A}}\chi_A$ is not a Lorentz scalar. The field that transforms with $\tilde{s}^{-1}$ is

$$\chi^A = \epsilon^{AB}\chi_B \tag{2.4}$$

where

$$(\epsilon^{AB}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2.5}$$

Thus, $\chi^A\psi_A$ is a Lorentz scalar, but $\chi^A\chi_A$ and $\chi^A{}_{,\mu}\,\chi_{A,\mu}$ vanish identically. The above considerations explain why the Klein-Gordon equation cannot be derived from a Lagrangian density in the usual manner. This equation is equivalent to the Dirac equation,[1] which can be obtained from a Lagrangian density, but this involves a variation of four independent fields, and a substitution leads to a third-order equation. The conserved current density expressed in terms of the two-component spinors is

$$j_\mu^{(\mathrm{D})} = \chi^*_{\dot{A}}\,\sigma_\mu^{\dot{A}B}\chi_B + \chi^*_{\dot{A},\alpha}\sigma_\alpha^{\dot{A}B}\sigma_{\mu\dot{C}B}\sigma_\beta^{\dot{C}D}\chi_{D,\beta}/m^2 \tag{2.6}$$

where $(\sigma_0^{\dot{A}B})$ is a unit matrix and the $(\sigma_i^{\dot{A}B})$ are the Pauli matrices. The charge density $j_0^{(\mathrm{D})}$ is positive definite, and cannot be used in our version of relativistic quantum mechanics (Marx, 1970a). There is another conserved real vector (we multiply by $-e$, the charge of the electron)

$$j_\mu = -ie(\chi^A\chi_{A,\mu} - \chi^{*\dot{A}}\chi^*_{\dot{A},\mu}) \tag{2.7}$$

which satisfies the conservation law

$$j_{\mu,\mu} = 0 \tag{2.8}$$

as a consequence of equations (2.1) and (2.4), and j_0 is not negative definite.

We decompose the solutions of equation (2.1) in momentum space in terms of helicity amplitudes, and find

$$\chi_A(x) = (2\pi)^{-3/2}\int d^3p(2p_0)^{-1/2}\sum_\lambda [b_\lambda(\mathbf{p})\chi_A{}^\lambda(\hat{p})\exp(-ip\cdot x) + d^*_\lambda(\mathbf{p})\chi_A^{-\lambda}(\hat{p})\exp(ip\cdot x)] \tag{2.9}$$

where $\hat{p}$ is a unit vector in the direction of $\mathbf{p}$ defined by its polar coordinates θ and ϕ

$$p_0 = (\mathbf{p}^2 + m^2)^{1/2} \tag{2.10}$$

[1] See Marx (1974) and the references found therein.

and

$$\chi_A^+ = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\exp(i\phi) \end{pmatrix}, \qquad \chi_A^- = \begin{pmatrix} -\sin(\theta/2)\exp(-i\phi) \\ \cos(\theta/2) \end{pmatrix} \tag{2.11}$$

We compute the total charge

$$Q = -ie \int d^3x(\chi^A \dot{\chi}_A - \chi^{*\dot{A}} \dot{\chi}_{\dot{A}}^*) \tag{2.12}$$

where the dot on the χ indicates a derivative with respect to time, and obtain

$$Q = -e \int d^3p[b_+(\mathbf{p})\, d_+^*(\mathbf{p}) - b_-(\mathbf{p})\, d_-^*(\mathbf{p}) + b_+^*(\mathbf{p})\, d_+(\mathbf{p}) - b_-^*(\mathbf{p})\, d_-(\mathbf{p})] \tag{2.13}$$

where we have used

$$\epsilon^{AB}\chi_B^+(\hat{p})\chi_A^+(\hat{p}) = \epsilon^{AB}\chi_B^-(\hat{p})\chi_A^-(\hat{p}) = 0 \tag{2.14}$$

$$\epsilon^{AB}\chi_B^+(\hat{p})\chi_A^-(\hat{p}) = -\epsilon^{AB}\chi_B^-(\hat{p})\chi_A^+(\hat{p}) = -1 \tag{2.15}$$

$$\epsilon^{AB}\chi_B^+(\hat{p})\chi_A^+(-\hat{p}) = \exp(i\phi),\ \epsilon^{AB}\chi_B^-(\hat{p})\chi_A^-(-\hat{p}) = \exp(-i\phi) \tag{2.16}$$

$$\epsilon^{AB}\chi_B^+(\hat{p})\chi_A^-(-\hat{p}) = \epsilon^{AB}\chi_B^-(\hat{p})\chi_A^+(-\hat{p}) = 0 \tag{2.17}$$

We define the new amplitudes

$$a_\lambda(\mathbf{p}) = [b_\lambda(\mathbf{p}) + \lambda\, d_\lambda(\mathbf{p})]/\sqrt{2} \tag{2.18}$$

$$c_\lambda(\mathbf{p}) = [-\lambda b_\lambda(\mathbf{p}) - d_\lambda(\mathbf{p})]/\sqrt{2} \tag{2.19}$$

and the charge becomes

$$Q = -e \int d^3p \sum_\lambda [|a_\lambda(\mathbf{p})|^2 - |c_\lambda(\mathbf{p})|^2] \tag{2.20}$$

This expression suggests that we interpret a_+ and a_- as probability amplitudes for particles (electrons), and c_+ and c_- as those for antiparticles (positrons). Equation (2.9) implies that

$$a_\lambda(\mathbf{p}, t) = a_\lambda(\mathbf{p}) \exp(-ip_0 t) \tag{2.21}$$

$$c_\lambda(\mathbf{p}, t) = c_\lambda(\mathbf{p}) \exp(-ip_0 t) \tag{2.22}$$

which shows that the probability densities in momentum space, $|a_\lambda(p, t)|^2$ and $|c_\lambda(p, t)|^2$, are constant. We can define the corresponding probability amplitudes in position space, guided by previous experience and equations (3.11) and (3.12) below, by

$$g_A^{(e)}(x) = (2\pi)^{-3/2} \int d^3p \left[\sum_\lambda a_\lambda(\mathbf{p}, t)\chi_A{}^\lambda(\hat{p}) \right] \exp(i\mathbf{p}\cdot\mathbf{x}) \tag{2.23}$$

$$g_A^{(p)}(x) = (2\pi)^{-3/2} \int d^3p \left[\sum_\lambda c_\lambda^*(\mathbf{p}, t)\chi_A{}^\lambda(\hat{p}) \right] \exp(-i\mathbf{p}\cdot\mathbf{x}) \tag{2.24}$$

and the probability densities, summed over spin states, are

$$\rho^{(e,p)}(x) = g^{(e,p)\dagger}(x)g^{(e,p)}(x) \tag{2.25}$$

We can consequently express the total charge in the form

$$Q = -e\int d^3x\,[\rho^{(e)}(x) - \rho^{(p)}(x)] \tag{2.26}$$

the difference between the electron and positron terms.

The Klein-Gordon equation (2.1) is covariant under proper orthochronous Lorentz transformations, that is, under a simultaneous change in the coordinates by

$$x'_\mu = a_\mu{}^\nu x_\nu \tag{2.27}$$

where $a_\mu{}^\nu$ satisfies

$$a_\mu{}^\lambda a_\nu{}^\rho g_{\lambda\rho} = g_{\mu\nu} \tag{2.28}$$

$$\det(a_\mu{}^\nu) = 1, \qquad a_0{}^0 > 0 \tag{2.29}$$

and the field according to equation (2.2), where $s_A{}^B$ is related to $a_\mu{}^\nu$ by

$$s_A{}^B s_{\dot{C}}^{*\dot{D}} a_\mu{}^\nu \sigma_{\nu\dot{D}B} = \sigma_{\mu\dot{C}A} \tag{2.30}$$

Equation (2.1) is also invariant under spatial reflection of coordinates, time reflection, and complex conjugation. This means that if $\chi_A(\mathbf{x}, t)$ is a solution of this equation, so are

$$\chi'_A(\mathbf{x}, t) = \chi_A(-\mathbf{x}, t) \tag{2.31}$$

$$\chi''_A(\mathbf{x}, t) = \chi_A(\mathbf{x}, -t) \tag{2.32}$$

and

$$\chi'''_A(\mathbf{x}, t) = [\chi^A(\mathbf{x}, t)]^* \tag{2.33}$$

From the momentum-space expansion (2.9), we find that

$$\chi'_A(x) = (2\pi)^{-3/2}\int d^3p(2p_0)^{-1/2}\sum_\lambda [b_\lambda(-\mathbf{p})\chi_A{}^\lambda(-\hat{p})\,\exp(-ip\cdot x) + d_\lambda^*(-\mathbf{p})\chi_A^{-\lambda}(-\hat{p})\exp(ip\cdot x)] \tag{2.34}$$

and, since

$$\chi_A{}^\lambda(-\hat{p}) = -\lambda\,\exp(i\lambda\phi)\chi_A^{-\lambda}(\hat{p}) \tag{2.35}$$

we have

$$a'_\lambda(\mathbf{p}) = \lambda\,\exp(-i\phi)a_{-\lambda}(-\mathbf{p}) \tag{2.36}$$

$$c'_\lambda(\mathbf{p}) = -\lambda\,\exp(-i\phi)c_{-\lambda}(-\mathbf{p}) \tag{2.37}$$

According to our interpretation of probability amplitudes, particle amplitudes become those of opposite momentum and helicity, and antiparticle amplitudes do likewise (aside from a simple phase factor). Under time reflection, we find

$$\chi_A''(x) = (2\pi)^{-3/2} \int d^3p\,(2p_0)^{-1/2} \sum_\lambda [b_\lambda(-\mathbf{p})\chi_A{}^\lambda(-\hat{p})\exp(ip\cdot x) + d_\lambda^*(-\mathbf{p})\chi_A^{-\lambda}(-\hat{p})\exp(-ip\cdot x)] \tag{2.38}$$

$$a_\lambda''(\mathbf{p}) = \lambda \exp(-i\lambda\phi) c_\lambda^*(-\mathbf{p}) \tag{2.39}$$

$$c_\lambda''(\mathbf{p}) = -\lambda \exp(-i\lambda\phi) a_\lambda^*(-\mathbf{p}) \tag{2.40}$$

Thus, particle amplitudes become antiparticle amplitude of opposite momentum and the same helicity, and vice versa (aside from phase factors and complex conjugation). Similarly,

$$\chi_A'''(x) = (2\pi)^{-3/2} \int d^3p(2p_0)^{-1/2} \sum_\lambda [b_\lambda^*(\mathbf{p})\chi^{\lambda*\dot{A}}(\hat{p})\exp(ip\cdot x) + d_\lambda(\mathbf{p})\chi^{-\lambda*\dot{A}}(\hat{p})\exp(-ip\cdot x)] \tag{2.41}$$

and we use

$$\chi^{+*\dot{A}}(\hat{p}) = \begin{pmatrix} \sin(\theta/2)\exp(-i\phi) \\ -\cos(\theta/2) \end{pmatrix}, \chi^{-*\dot{A}}(\hat{p}) = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\exp(i\phi) \end{pmatrix} \tag{2.42}$$

to derive

$$a_\lambda'''(\mathbf{p}) = \lambda c_\lambda(\mathbf{p}) \tag{2.43}$$

$$c_\lambda'''(\mathbf{p}) = -\lambda a_\lambda(\mathbf{p}) \tag{2.44}$$

Under this transformation, particle and antiparticle amplitudes are exchanged without change in momentum or helicity, that is, we interpret it as charge conjugation. A combination of time reflection and charge conjugation results in Wigner time reversal.

Although we do not have tensors that can be interpreted as stress-energy and angular momentum densities, we can define the conserved quantities in terms of the probability amplitudes, such as

$$P_\mu = \int d^3p\, p_\mu \sum_\lambda (|a_\lambda|^2 + |c_\lambda|^2) \tag{2.45}$$

for the energy-momentum vector, and

$$\mathbf{S} = \int d^3x\,[g^{(e)\dagger}\boldsymbol{\sigma}g^{(e)} + g^{(p)\dagger}\boldsymbol{\sigma}g^{(p)}] \tag{2.46}$$

for the spin.

3. *The Inhomogeneous Equation*

We now consider the solution of the Klein–Gordon equation with a given source and the specification of "boundary" conditions at the initial and final times.

The probability amplitudes are a peculiar mixture of positive- and negative-frequency parts of the free field. However, we can still use the causal Green function $\Delta_F(x)$ to solve the inhomogeneous equation when particle amplitudes are specified at the initial time and antiparticle amplitudes at the final time, as we do in relativistic quantum mechanics.

The equation we have to solve is

$$(\partial^2 + m^2)\chi_A(x) = \omega_A(x) \tag{3.1}$$

and the Green function satisfies

$$(\partial^2 + m^2)\Delta_F(x) = -\delta(x) \tag{3.2}$$

We can express Δ_F in terms of the step function $\theta(t)$ and the solutions $\Delta^{(\pm)}(x)$ of the homogeneous Klein-Gordon equation by

$$\Delta_F(x) = \theta(t)\Delta^{(+)}(x) - \theta(-t)\Delta^{(-)}(x) \tag{3.3}$$

We substitute

$$\phi^B(x') = \delta_A{}^B \Delta_F(x' - x) \tag{3.4}$$

in Green's theorem

$$\begin{aligned}\int d^4x' \, [\phi^B(x')(\partial'^2 + m^2)\chi_B(x') - \chi_B(x')(\partial'^2 + m^2)\phi^B(x')] \\ = \int d^3x' [\phi^B(x')\dot{\chi}_B(x') - \chi_B(x')\dot{\phi}^B(x')]_{t'=t_i}^{t'=t_f}\end{aligned} \tag{3.5}$$

and use

$$\partial_0 \Delta^{(\pm)}(x) = \mp i\tilde{E}\Delta^{(\pm)}(x) \tag{3.6}$$

where $\tilde{E}$ is the integral operator

$$\tilde{E} = (-\nabla^2 + m^2)^{1/2} \tag{3.7}$$

We integrate by parts and obtain

$$\begin{aligned}\chi_A(x) = -\int d^3x' \Bigg\{ \int_{t_i}^{t_f} dt' \Delta_F(x' - x)\omega_A(x') \\ + i[\Delta^{(+)}(\mathbf{x} - \mathbf{x}', t - t_i)(2\tilde{E}')(\tfrac{1}{2} + \tfrac{1}{2}i\tilde{E}'^{-1}\partial_0')\chi_A(\mathbf{x}', t_i) \\ - \Delta^{(-)}(\mathbf{x} - \mathbf{x}', t - t_f)(2\tilde{E}')(\tfrac{1}{2} - \tfrac{1}{2}i\tilde{E}'^{-1}\partial_0')\chi_A(\mathbf{x}', t_f)] \Bigg\}\end{aligned} \tag{3.8}$$

The combinations of the field and the time derivative of the field correspond to the positive- and negative-frequency parts for a field that is free at the initial and final times, and we define

$$\chi_A^{(\pm)}(x) = \tfrac{1}{2}(1 \pm i\tilde{E}^{-1}\partial_0)\chi_A(x) \tag{3.9}$$

But this separation does not correspond to particles and antiparticles in the present approach, and we have to modify equation (3.8). Applying Green's theorem to $\chi^{*\dot{A}}$ and Δ_F and adding the terms to those in equation (3.8), we obtain

$$\chi_A(x) + \chi^{*\dot{A}}(x) = -\int d^3x' \left\{ \int_{t_i}^{t_f} dt' \Delta_F(x' - x)[\omega_A(x') + \omega^{*\dot{A}}(x')] \right. \\ + i\Delta^{(+)}(\mathbf{x} - \mathbf{x}', t - t_i)(2\tilde{E}')[\chi_A^{(+)}(\mathbf{x}', t_i) + \chi^{*(+)\dot{A}}(\mathbf{x}', t_i)] \\ \left. - i\Delta^{(-)}(\mathbf{x} - \mathbf{x}', t - t_f)(2\tilde{E}')[\chi_A^{(-)}(\mathbf{x}', t_f) + \chi^{*(-)\dot{A}}(\mathbf{x}', t_f)] \right\} \tag{3.10}$$

The left-hand side takes the values $\chi_1 + (\chi_2)^*$ and $\chi_2 - (\chi_1)^*$, which determine χ_1 and χ_2. Furthermore, we use equations (2.42) to reduce the solutions of the homogeneous equation to

$$\chi_A^{(+)}(x) + \chi^{*(+)\dot{A}}(x) = (2\pi)^{-3/2} \int d^3p\, p_0^{-1/2} \left[\sum_\lambda a_\lambda(\mathbf{p}) \chi_A{}^\lambda(\hat{p}) \right] \\ \times \exp(-ip \cdot x) \tag{3.11}$$

$$\chi_A^{(-)}(x) + \chi^{*(-)\dot{A}}(x) = (2\pi)^{-3/2} \int d^3p\, p_0^{-1/2} \left[\sum_\lambda c_\lambda^*(\mathbf{p}) \chi_A{}^\lambda(\hat{p}) \right] \\ \times \exp(ip \cdot x) \tag{3.12}$$

which are precisely the amplitudes that should be specified at t_i and t_f, respectively.

If the source ω_A is an interaction term, we can use equation (3.10) to compute the terms in a perturbation expansion of the solution.

4. *The Massless Field*

If we set the mass equal to zero in the Klein-Gordon equation (2.1), we obtain d'Alembert's equation

$$\partial^2 \chi_A(x) = 0 \tag{4.1}$$

Another equation that is frequently used for a massless spinor field is the first-order Weyl equation

$$\partial^{\dot{B}A} \chi_A(x) = 0 \tag{4.2}$$

or

$$\dot{\chi} + \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \chi = 0 \tag{4.3}$$

The solutions of equation (4.1) can still be written in the form (2.9); those that also obey equation (4.2) are further restricted by

$$\int d^3p (2p_0)^{1/2} [b_-(\mathbf{p}) \chi_A^-(\hat{p}) \exp(-ip \cdot x) - d_+^*(\mathbf{p}) \chi_A^-(\hat{p}) \exp(ip \cdot x)] = 0 \tag{4.4}$$

since we can set

$$p_0 = |\mathbf{p}| \tag{4.5}$$

$$\boldsymbol{\sigma} \cdot \hat{p}\chi^\lambda(\hat{p}) = \lambda\chi^\lambda(\hat{p}) \tag{4.6}$$

Consequently,

$$b_-(\mathbf{p}) = d_+(\mathbf{p}) = 0 \tag{4.7}$$

and, from equation (2.13),

$$Q = 0 \tag{4.8}$$

We can also come to this conclusion by substituting $\dot{\chi}_1$ and $\dot{\chi}_2$ in the charge density j_0 from equation (4.3); the resulting expression is a combination of spatial derivatives that do not contribute to an integral over all space.

States with well-defined helicity are superpositions of particle and antiparticle amplitudes.

5. *Electromagnetic Interactions*

We have not found a good way to introduce electromagnetic interactions between the classical spinor and electromagnetic fields. We do not have a Lagrangian density to start from, and the "gauge-invariant" substitution

$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu \tag{5.1}$$

is useful only when the charged field transforms according to

$$\chi'_A(x) = \exp\,[ie\Lambda(x)]\,\chi_A(x) \tag{5.2}$$

under gauge transformations of the second kind. But the current density (2.7) is not invariant under gauge transformations of the first kind (with a constant Λ), and the relation (5.2) is not acceptable. Consequently, the two-component spinor form of the Dirac equation,

$$[(D^2 + m^2)\delta_B{}^A + ieF_{\mu\nu}\mathscr{S}_{\mu\nu B}{}^A]\,\chi_A(x) = 0 \tag{5.3}$$

where

$$\mathscr{S}_{\mu\nu B}{}^A = \tfrac{1}{4}(\sigma_{\mu\dot{C}B}\sigma_\nu^{\dot{C}A} - \sigma_{\nu\dot{C}B}\sigma_\mu^{\dot{C}A}) \tag{5.4}$$

is no longer gauge invariant. Also, the current density (2.7) is no longer conserved. This is not surprising considering that it differs from the usual conserved current density for the Dirac equation, which has a positive definite probability density.

Many interaction terms can be added to the Klein-Gordon equation, gauge invariant or not, involving either χ_A or χ_A^*, and a similar number that can be added to the current density (2.7). We have not found a good way to choose a particular combination.

Another approach that might be explored further is that of transformations in spinor space that are functions of space and time, along the lines that led to the Yang–Mills field in the case of isospinor space. The connection between the basis in spinor space and the reference frame is based on our choice of $\sigma_\mu^{\dot{A}B}$, and a unimodular transformation, determined up to a sign, is induced in spinor space by a proper orthochronous Lorentz transformation. Other relationships between the two spaces might also be useful.

An alternative to an interaction between the fields is an independent formulation of relativistic quantum mechanics based on the free-field theory and the classical interaction between charged particles.

6. *Concluding Remarks*

We have used an unfamiliar conserved Lorentz vector for the free two-component classical spinor field to define a charge that can take negative as well as positive values, and have defined probability amplitudes in agreement with the relativistic quantum mechanics of scalar particles.

When the mass becomes zero, we can restrict the solutions to those of the Weyl equation. In this case, the charge that we have defined vanishes, providing a possible reason for the absence of charged massless spin-$\frac{1}{2}$ particles in nature.

It is difficult to find interactions that allow for a conserved current density that reduces to the one we have defined here when the coupling constant vanishes. We intend to investigate this question in the future, and to apply new ideas from classical electrodynamics to relativistic quantum mechanics.

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